The first four lessons of this chapter examined situations in which all voters are considered equals. This lesson examines situations in which some voters have more votes than other voters.

**Public Hearing Set for Madison County Weighted Voting**

*Oneida Daily Dispatch*
*April 11, 2013*

The Madison County Board of Supervisors will hear public comments over upgrading weighted voting totals.

In Madison County, supervisors vote based on a weighted formula calculated by the population of their towns. That voting configuration hasn’t been updated since 2002. Using population data from the 2010 Census, the number has been recalculated.

Under the proposed law, there are to be 1500 votes total. The votes are not divisible and are to be cast as one lump sum by each town. The county uses the Banzhaf power index to determine how many votes each supervisor has.
Members of legislative bodies such as the United States Congress and county boards represent districts. The constitutional principle of one person, one vote requires that such districts be approximately the same size. Thus, every 10 years, district lines are redrawn to reflect changes in population. But redrawing boundaries can be difficult for many reasons. Some legislative bodies try to resolve difficulties by adopting a system in which some votes carry more weight than others.

Consider a simple example. A small high school has 110 students. Because of recent growth in the size of the community, the sophomore class is quite large. It has 50 members, and the junior and senior classes each have 30 members.

The school’s student council is composed of a single representative from each class. Each of the three members is given a number of votes proportionate to the size of the class represented. Accordingly, the sophomore representative has five votes, and the junior and senior representatives each have three. The passage of any issue that is before the council requires a simple majority of six votes.

The student council’s voting model is an example of weighted voting. Weighted voting occurs whenever some members of a voting body have more votes than others have.

In recent years, several people have questioned whether weighted voting is fair. Among them is John Banzhaf III, a law professor at George Washington University who has initiated several legal actions against weighted voting procedures used in local government.

To understand Banzhaf’s objection to weighted voting, consider the number of ways that voting on an issue could occur in the student council example.

It is possible that an issue is favored by none of the members, one of them, two of them, or all three. In which cases does an issue pass? The following list gives all possible ways of voting for an issue and the associated number of votes.

\[
\{; 0\} \{\text{So}; 5\} \{\text{Jr}; 3\} \{\text{Sr}; 3\} \{\text{So, Jr}; 8\} \{\text{So, Sr}; 8\} \{\text{Jr, Sr}; 6\} \{\text{So, Jr, Sr}; 11\}
\]

For example, \{\text{Jr, Sr}; 6\} indicates that the junior and senior representatives vote for an issue and that they have a total of six votes between them.
Each of these collections of voters is called a coalition. Those with enough votes to pass an issue are winning coalitions. The winning coalitions in this example are those with six or more votes and are listed below along with their respective vote totals.

\[\{\text{So, Jr; 8}\} \{\text{So, Sr; 8}\} \{\text{Jr, Sr; 6}\} \{\text{So, Jr, Sr; 11}\}\]

The last winning coalition is different from the other three in one important respect: If any one of the members decides to vote differently, the coalition still wins. No single member is essential to the coalition. Banzhaf argued that the only time a voter has power is when the voter belongs to a coalition that needs the voter in order to pass an issue. The coalitions for which at least one member is essential are

\[\{\text{So, Jr; 8}\} \{\text{So, Sr; 8}\} \{\text{Jr, Sr; 6}\}.\]

Notice that the sophomore representative is essential to two of the coalitions, which is also true of the junior and senior representatives. In other words, about the same number of times, each of the representatives can be expected to cast a key vote in passing an issue.

A paradox: Although the votes have been distributed to give greater power to the sophomores, the outcome is that all members have the same power!

Since distributing the votes in a way that reflects the population distribution does not always result in a fair distribution of power, mathematical procedures can be used to develop ways to measure actual power in weighted voting situations.

A measure of the power of a member of a voting body is called a power index. In this lesson, a voter’s power index is the number of winning coalitions in which the voter is essential. For example, in the student council situation, the sophomore representative is essential to two winning coalitions and thereby has a power index of 2, as do the junior and senior representatives.
Lesson 1.5 • Weighted Voting and Voting Power

A Power Index Algorithm

1. List all coalitions of voters that are winning coalitions.
2. Select any voter, and record a 0 for that voter’s power index.
3. From the list in step 1, select a coalition of which the voter selected in step 2 is a member. Subtract the number of votes the voter has from the coalition’s total. If the result is less than the number of votes required to pass an issue, add 1 to the voter’s power index.
4. Repeat step 3 until all coalitions of which the voter chosen in step 2 is a member are checked.
5. Repeat steps 2 through 4 until all voters are checked.

Exercises

1. Consider a situation in which A, B, and C have 3, 2, and 1 votes respectively, and in which 4 votes are required to pass an issue.
   a. List all possible coalitions and all winning coalitions.
   b. Determine a power index for each voter.
   c. If the number of votes required to pass an issue is increased from 4 to 5, determine a power index for each voter.
2. In a situation with three voters, A has 7 votes, B has 3, and C has 3. A simple majority is required to pass an issue.
   a. Determine a power index for each voter.
   b. A dictator is a member of a voting body who has all the power. A dummy is a member who has no power. Are there any dictators or dummies in this situation?
3. The student council example in this lesson depicts a situation with three voters that results in equal power for all three. In Exercises 1 and 2, power is distributed differently. Find a distribution of votes that results in a power distribution among three voters that is different from the ones you have already seen. How many new power distributions in situations with three voters can you find?
4. In this lesson's student council example, can the votes be distributed so that the members' power indices are proportionate to the class sizes? Explain.

5. In this lesson's student council example, suppose that the representatives of the junior and senior classes always differ on issues and never vote alike. Does this make any practical difference in the power of the three representatives? Explain.

6. (See Exercise 14 of Lesson 1.4 on page 33.) Let $C_n$ represent the number of coalitions that can be formed in a group of $n$ voters. Write a recurrence relation that describes the relationship between $C_n$ and $C_{n-1}$.

7. One way to determine all winning coalitions in a weighted voting situation is to work from a list of all possible coalitions. Use A, B, C, and D to represent the individuals in a group of four voters and list all possible coalitions.

8. Weighted voting is commonly used to decide issues at meetings of corporate stockholders. Each member has one vote for each share of stock held.

   a. A company has four stockholders: A, B, C, and D. They own 26%, 25%, 25%, and 24% of the stock, respectively, and more than 50% of the vote is needed to pass an issue. Determine a power index for each stockholder. (Use your results from Exercise 7 as an aid.)

   b. Another company has four stockholders. They own 47%, 41%, 7%, and 5% of the stock. Find a power index for each stockholder.

   c. Compare the percentage of stock owned by the smallest shareholder in parts a and b. Do the same for the power index of the smallest stockholder in each case.
9. A landmark court decision on voting power involved the Nassau County, New York, Board of Supervisors. In 1964, the board had six members. The number of votes given to each was 31, 31, 21, 28, 2, and 2.

a. Determine a power index for each member.

b. The board was composed of representatives of five municipalities with these populations:

<table>
<thead>
<tr>
<th>Municipality</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hempstead</td>
<td>728,625</td>
</tr>
<tr>
<td>North Hempstead</td>
<td>213,225</td>
</tr>
<tr>
<td>Oyster Bay</td>
<td>285,545</td>
</tr>
<tr>
<td>Glen Cove</td>
<td>22,752</td>
</tr>
<tr>
<td>Long Beach</td>
<td>25,654</td>
</tr>
</tbody>
</table>

The members with 31 votes represented Hempstead. The others each represented the municipality listed in the same order as in the table. Compare the power indices of the municipalities with their populations.

10. A minimal winning coalition is one in which all voters are essential.

a. Give an example of a weighted voting situation with a winning coalition for which at least one but not all of the voters is essential. Identify the minimal winning coalitions in this situation.

b. Is defining a voter’s power index as the number of minimal winning coalitions to which the voter belongs equivalent to the definition used in this lesson? Explain.
11. The president of the United States is chosen in the Electoral College, a system that can be considered a form of weighted voting among the states. The number of electors given to each state (and the District of Columbia) is equal to its representation in Congress. That is, the number of electors equals the number of members of the House of Representatives plus two (the number of senators). In 2000, Albert Gore won the popular vote by about half a million votes over George Bush, but lost in the Electoral College.

a. In the 2000 census, the population of California (the most populous state) was 33,871,648. The population of Wyoming (the least populous state) was 493,782. California has 52 representatives in the U.S. House. Wyoming has 1. Use these data to construct an argument that the electoral vote distribution is weighted in favor of small states.

b. Some reformers have proposed removing the electoral weighting. They suggest equating the number of electors with the membership in the U.S. House only. The following table shows the Electoral College votes in the 2000 election. Would this reform proposal change the 2000 election results? Explain.

<table>
<thead>
<tr>
<th></th>
<th>Bush</th>
<th>Gore</th>
<th></th>
<th>Bush</th>
<th>Gore</th>
<th></th>
<th>Bush</th>
<th>Gore</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama</td>
<td>9</td>
<td></td>
<td>Kentucky</td>
<td>8</td>
<td></td>
<td>North Dakota</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Alaska</td>
<td>3</td>
<td></td>
<td>Louisiana</td>
<td>9</td>
<td></td>
<td>Ohio</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Arizona</td>
<td>8</td>
<td></td>
<td>Maine</td>
<td>4</td>
<td></td>
<td>Oklahoma</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Arkansas</td>
<td>6</td>
<td></td>
<td>Maryland</td>
<td>10</td>
<td></td>
<td>Oregon</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>California</td>
<td>54</td>
<td></td>
<td>Massachusetts</td>
<td>12</td>
<td></td>
<td>Pennsylvania</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Colorado</td>
<td>8</td>
<td></td>
<td>Michigan</td>
<td>18</td>
<td></td>
<td>Rhode Island</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Connecticut</td>
<td>8</td>
<td></td>
<td>Minnesota</td>
<td>10</td>
<td></td>
<td>South Carolina</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Delaware</td>
<td>3</td>
<td></td>
<td>Mississippi</td>
<td>7</td>
<td></td>
<td>South Dakota</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>D.C.</td>
<td>3</td>
<td></td>
<td>Missouri</td>
<td>11</td>
<td></td>
<td>Tennessee</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Florida</td>
<td>25</td>
<td></td>
<td>Montana</td>
<td>3</td>
<td></td>
<td>Texas</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Georgia</td>
<td>13</td>
<td></td>
<td>Nebraska</td>
<td>5</td>
<td></td>
<td>Utah</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Hawaii</td>
<td>4</td>
<td></td>
<td>Nevada</td>
<td>4</td>
<td></td>
<td>Vermont</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Idaho</td>
<td>4</td>
<td></td>
<td>New Hampshire</td>
<td>4</td>
<td></td>
<td>Virginia</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Illinois</td>
<td>22</td>
<td></td>
<td>New Jersey</td>
<td>15</td>
<td></td>
<td>Washington</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Indiana</td>
<td>12</td>
<td></td>
<td>New Mexico</td>
<td>5</td>
<td></td>
<td>West Virginia</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Iowa</td>
<td>7</td>
<td></td>
<td>New York</td>
<td>33</td>
<td></td>
<td>Wisconsin</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Kansas</td>
<td>6</td>
<td></td>
<td>North Carolina</td>
<td>14</td>
<td></td>
<td>Wyoming</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
c. The Electoral College is mandated in the United States Constitution. Why do you think the founders of the country instituted a system weighted in favor of small states? (Do some research if you are not sure.)

Computer Explorations

12. Use the weighted voting software that accompanies this book to experiment with different weighted voting systems when there are three voters. Change the number of votes given to each voter and the number of votes required to pass an issue. How many different power distributions are possible? Do the same for weighted voting systems with four voters.

Projects

13. The Security Council of the United Nations is composed of five permanent members and ten others who are elected to two-year terms. For a measure to pass, it must receive at least nine votes that include all five of the permanent members. Determine a power index for a permanent member and for a temporary member. (Assume that all members are present and voting.)

European Voice
April 29, 2013

A majority of member states today voted to approve a European Commission proposal to ban the use of neonicotinoid seed treatment pesticides, thought to be harmful to bees.

The outcome was close, following a first vote last month which was inconclusive, with neither a majority for or against. Crucially, Germany changed its position in today’s vote to approve the ban, after having abstained last month. The 15 member states voting in favor gave 189 weighted votes, versus 125 weighted votes against, including those of the UK. Abstentions amounted to 33 weighted votes.
14. Research and report on other power indices. What, for example, is the Shapley-Shubik power index?

15. What is the effect of the Electoral College system on the power of individual voters in selecting the president? Research the matter and report on the relative power of voters in different states.

16. Research and report on court decisions about weighted voting.

---

**U.K., U.S. Wield Most Cyber Power**

*National Journal*

January 13, 2012

The United Kingdom and United States lead other developing countries in their ability to withstand cyberattacks and develop strong digital economies, according to a new study by Booz Allen Hamilton and the Economist Intelligence Unit.

The U.K. tops the rest of the Group of 20 nations, including the U.S., in the "Cyber Power Index." The European Union, considered the 20th member of the G20, was not included.

The index, which put the U.S. in the No. 2 spot, rates the countries on their legal and regulatory frameworks; economic and social issues; technology infrastructure; and industry.

"Overall, the top five countries exhibiting cyber power, as measured by the index--the U.K.; the U.S.; Australia; Germany; and Canada--illustrate that developed Western countries are leading the way into the digital era," the companies said in a statement.