Simplifying, Multiplying, and Dividing Rational Expressions

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CHAPTER 1 Simplifying, Multiplying, and Dividing Rational Expressions

Objective

To simplify, multiply, and divide rational expressions.

Review Queue

Simplify the following fractions.

1. $\frac{8}{20}$
2. $\frac{6x^3y^2}{9xy^5}$
3. $\frac{7a^5bc^2}{35ab^4c^9}$

Multiply or divide the following fractions.

4. $\frac{4}{5} \cdot \frac{10}{18}$
5. $\frac{2}{3} \div \frac{1}{4}$
6. $\frac{12}{5} \div \frac{3}{10}$

Simplifying Rational Expressions

Objective

To simplify rational expressions involving factorable polynomials.

Guidance

Recall that a rational function is a function, $f(x)$, such that $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are both polynomials. A **rational expression**, is just $\frac{p(x)}{q(x)}$. Like any fraction, a rational expression can be simplified. To simplify a rational expression, you will need to factor the polynomials, determine if any factors are the same, and then cancel out any like factors.

Fraction: $\frac{9}{15} = \frac{\cancel{3} \cdot 3}{\cancel{3} \cdot 5} = \frac{3}{5}$

Rational Expression: $\frac{x^2+6x+9}{x^2+8x+15} = \frac{\cancel{(x+3)}(x+3)}{\cancel{(x+3)}(x+5)} = \frac{x+3}{x+5}$

With both fractions, we broke apart the numerator and denominator into the prime factorization. Then, we canceled the common factors.

Important Note: $\frac{x+3}{x+5}$ is completely factored. ***Do not*** cancel out the x's! $\frac{3x}{5x}$ reduces to $\frac{3}{5}$, but $\frac{x+3}{x+5}$ does not because of the addition sign. To prove this, we will plug in a number for x to and show that the fraction does not reduce to $\frac{3}{5}$. If $x = 2$, then $\frac{2+3}{2+5} = \frac{5}{7} \neq \frac{3}{5}$.

Example A

Simplify $\frac{2x^3}{4x^2-6x}$.

Solution: The numerator factors to be $2x^3 = 2 \cdot x \cdot x \cdot x$ and the denominator is $4x^2 - 6x = 2x(2x-3)$.

$$\frac{2x^3}{4x^2-6x} = \frac{\cancel{2} \cdot \cancel{x} \cdot x \cdot x}{\cancel{2} \cdot \cancel{x} \cdot (2x-3)} = \frac{x^2}{2x-3}$$

Example B

Simplify $\frac{6x^2-7x-3}{2x^3-3x^2}$.

Solution: If you need to review factoring, see the *Factoring Quadratics when the Leading Coefficient is 1* concept and the *Factoring Quadratics when the Leading Coefficient is not 1* concept. Otherwise, factor the numerator and find the GCF of the denominator and cancel out the like terms.

$\frac{6x^2-7x-3}{2x^3-3x^2} = \frac{\cancel{(2x-3)}(3x+1)}{x^2\cancel{(2x-3)}} = \frac{3x+1}{x^2}$

Example C

Simplify $\frac{x^2-6x+27}{2x^2-19x+9}$.

Solution: Factor both the top and bottom and see if there are any common factors.

$\frac{x^2-6x+27}{2x^2-19x+9} = \frac{\cancel{(x-9)}(x+3)}{\cancel{(x-9)}(2x-1)} = \frac{x+3}{2x-1}$

Special Note: Not every polynomial in a rational function will be factorable. Sometimes there are no common factors. When this happens, write "not factorable."

Guided Practice

If possible, simplify the following rational functions.

1. $\frac{3x^2-x}{3x^2}$
2. $\frac{x^2+6x+8}{x^2+6x+9}$
3. $\frac{2x^2+x-10}{6x^2+17x+5}$
4. $\frac{x^3-4x}{x^5+4x^3-32x}$

Answers

1. $\frac{3x^2-x}{3x^2} = \frac{\cancel{x}(3x-1)}{3\cdot\cancel{x}\cdot x} = \frac{3x-1}{3x}$
2. $\frac{x^2+6x+8}{x^2+6x+9} = \frac{(x+4)(x+2)}{(x+3)(x+3)}$ There are no common factors, so this is reduced.
3. $\frac{2x^2+x-10}{6x^2+17x+5} = \frac{\cancel{(2x+5)}(x-2)}{\cancel{(2x+5)}(3x+1)} = \frac{x-2}{3x+1}$
4. In this problem, the denominator will factor like a quadratic once an x is pulled out of each term.

$\frac{x^3-4x}{x^5+4x^3-32x} = \frac{x(x^2-4)}{x(x^4+4x^2-32)} = \frac{x(x-2)(x+2)}{x(x^2-4)(x^2+8)} = \frac{\cancel{x(x-2)(x+2)}}{\cancel{x(x-2)(x+2)}(x^2+8)} = \frac{1}{x^2+8}$

Vocabulary

Rational Expression
A fraction with polynomials in the numerator and denominator.

Problem Set

Simplify the following Rational Expressions.

1. $\frac{4x^3}{2x^2+3x}$
2. $\frac{x^3+x^2-2x}{x^4+4x^3-5x^2}$
3. $\frac{2x^2-5x-3}{2x^2-7x-4}$
4. $\frac{5x^2+37x+14}{5x^3-33x^2-14x}$
5. $\frac{8x^2-60x-32}{-4x^2+26x+48}$
6. $\frac{6x^3-24x^2+30x-120}{9x^4+36x^2-45}$

7. $\frac{6x^2+5x-4}{6x^2-x-1}$
8. $\frac{x^4+8x}{x^4-2x^3+4x^2}$
9. $\frac{6x^4-3x^3-63x^2}{12x^2-84x}$
10. $\frac{x^5-3x^3-4x}{x^4+2x^3+x^2+2x}$
11. $\frac{-3x^2+25x-8}{x^3-8x^2+x-8}$
12. $\frac{-x^3+3x^2+13x-15}{-2x^3+7x^2+20x-25}$

Multiplying Rational Expressions

Objective

To multiply together two or more rational expressions and simplify.

Guidance

We take the previous concept one step further in this one and multiply two rational expressions together. When multiplying rational expressions, it is just like multiplying fractions. However, it is much, much easier to factor the rational expressions before multiplying because factors could cancel out.

Example A

Multiply $\frac{x^2-4x}{x^3-9x} \cdot \frac{x^2+8x+15}{x^2-2x-8}$

Solution: Rather than multiply together each numerator and denominator to get very complicated polynomials, it is much easier to first factor and then cancel out any common factors.

$\frac{x^2-4x}{x^3-9x} \cdot \frac{x^2+8x+15}{x^2-2x-8} = \frac{x(x-4)}{x(x-3)(x+3)} \cdot \frac{(x+3)(x+5)}{(x+2)(x-4)}$

At this point, we see there are common factors between the fractions.

$\frac{\cancel{x}\cancel{(x-4)}}{\cancel{x}(x-3)\cancel{(x+3)}} \cdot \frac{\cancel{(x+3)}(x+5)}{(x+2)\cancel{(x-4)}} = \frac{x+5}{(x-3)(x+2)}$

At this point, the answer is in **factored form** and simplified. You do not need to multiply out the base.

Example B

Multiply $\frac{4x^2y^5z}{6xyz^6} \cdot \frac{15y^4}{35x^4}$

Solution: These rational expressions are monomials with more than one variable. Here, we need to remember the laws of exponents from earlier concepts. Remember to add the exponents when multiplying and subtract the exponents when dividing. The easiest way to solve this type of problem is to multiply the two fractions together first and then subtract common exponents.

$\frac{4x^2y^5z}{6xyz^6} \cdot \frac{15y^4}{35x^4} = \frac{60x^2y^9z}{210x^5yz^6} = \frac{2y^8}{7x^3z^5}$

You can reverse the order and cancel any common exponents first and then multiply, but sometimes that can get confusing.

Example C

Multiply $\frac{4x^2+4x+1}{2x^2-9x-5} \cdot (3x-2) \cdot \frac{x^2-25}{6x^2-x-2}$

Solution: Because the middle term is a linear expression, rewrite it over 1 to make it a fraction.

$\frac{4x^2+4x+1}{2x^2-9x-5} \cdot (3x-2) \cdot \frac{x^2-25}{6x^2-x-2} = \frac{\cancel{(2x+1)}\cancel{(2x+1)}}{\cancel{(2x+1)}\cancel{(x-5)}} \cdot \frac{\cancel{3x-2}}{1} \cdot \frac{\cancel{(x-5)}(x+5)}{\cancel{(3x-2)}\cancel{(2x+1)}} = x+5$

Guided Practice

Multiply the following expressions.

1. $\frac{4x^2-8x}{10x^3} \cdot \frac{15x^2-5x}{x-2}$
2. $\frac{x^2+6x-7}{x^2-36} \cdot \frac{x^2-2x-24}{2x^2+8x-42}$
3. $\frac{4x^2y^7}{32x^4y^3} \cdot \frac{16x^2}{8y^6}$

Answers

1. $\frac{4x^2-8x}{10x^3} \cdot \frac{15x^2-5x}{x-2} = \frac{\cancel{2} \cdot 2\cancel{x}\cancel{(x-2)}}{\cancel{2} \cdot \cancel{5x} \cdot \cancel{x} \cdot x} \cdot \frac{\cancel{5x}(3x-1)}{\cancel{x-2}} = \frac{2(3x-1)}{x}$
2. $\frac{x^2+6x-7}{x^2-36} \cdot \frac{x^2-2x-24}{2x^2+8x-42} = \frac{\cancel{(x+7)}(x-1)}{\cancel{(x-6)}(x+6)} \cdot \frac{\cancel{(x-6)}(x+4)}{2\cancel{(x+7)}(x-3)} = \frac{(x-1)(x+4)}{2(x-3)(x+6)}$
3. $\frac{4x^2y^7}{32x^4y^3} \cdot \frac{16x^2}{8y^6} = \frac{64x^4y^7}{256x^4y^9} = \frac{1}{4y^2}$

Problem Set

Multiply the following expressions. Simplify your answers.

1. $\frac{8x^2y^3}{5x^3y} \cdot \frac{15xy^8}{2x^3y^5}$
2. $\frac{11x^3y^9}{2x^4} \cdot \frac{6x^7y^2}{33xy^3}$
3. $\frac{18x^3y^6}{13x^8y^2} \cdot \frac{39x^{12}y^5}{9x^2y^9}$
4. $\frac{3x+3}{y-3} \cdot \frac{y^2-y-6}{2x+2}$
5. $\frac{6}{2x+3} \cdot \frac{4x^2+4x-3}{3x+3}$
6. $\frac{6+x}{2x-1} \cdot \frac{x^2+5x-3}{x^2+5x-6}$
7. $\frac{3x-21}{x-3} \cdot \frac{-x^2+x+6}{x^2-5x-14}$
8. $\frac{6x^2+5x+1}{8x^2-2x-3} \cdot \frac{4x^2+28x-30}{6x^2-7x-3}$
9. $\frac{x^2+9x-36}{x^2-9} \cdot \frac{x^2+8x+15}{-x^2+11x+12}$
10. $\frac{2x^2+x-21}{x^2+2x-48} \cdot (4-x) \cdot \frac{2x^2-9x-18}{2x^2-x-28}$
11. $\frac{8x^2-10x-3}{4x^3+x^2-36x-9} \cdot \frac{5x+3}{x-1} \cdot \frac{x^3+3x^2-x-3}{5x^2+8x+3}$

Dividing Rational Expressions

Objective

To divide two or more rational expressions.

Guidance

Dividing rational expressions has one additional step than multiply them. Recall that when you divide fractions, you need to flip the second fraction and change the problem to multiplication. The same rule applies to dividing rational expressions.

Example A

Divide $\frac{5a^3b^4}{12ab^8} \div \frac{15b^6}{8a^6}$.

Solution: Flip the second fraction, change the $\div$ sign to multiplication and solve.

$\frac{5a^3b^4}{12ab^8} \div \frac{15b^6}{8a^6} = \frac{5a^3b^4}{12ab^8} \cdot \frac{8a^6}{15b^6} = \frac{40a^9b^4}{180ab^{14}} = \frac{2a^8}{9b^{10}}$

Example B

Divide $\frac{x^4-3x^2-4}{2x^2+x-10} \div \frac{x^3-3x^2+x-3}{x-2}$

Solution: Flip the second fraction, change the $\div$ sign to multiplication and solve.

$$\begin{aligned}\frac{x^4-3x^2-4}{2x^2+x-10} \div \frac{x^3-3x^2+x-3}{x-2} &= \frac{x^4-3x^2-4}{2x^2+x-10} \cdot \frac{x-2}{x^3-3x^2+x-3}\\ &= \frac{(x^2-4)(x^2+1)}{(2x-5)(x+2)} \cdot \frac{x-2}{(x^2+1)(x-3)}\\ &= \frac{(x-2)\cancel{(x+2)}\cancel{(x^2+1)}}{(2x-5)\cancel{(x+2)}} \cdot \frac{x-2}{\cancel{(x^2+1)}(x-3)}\\ &= \frac{(x-2)^2}{(2x-5)(x-3)}\end{aligned}$$

Review the *Factoring by Grouping* concept to factor the blue polynomial and the *Factoring in Quadratic Form* concept to factor the red polynomial.

Example C

Perform the indicated operations: $\frac{x^3-8}{x^2-6x+9} \div (x^2+3x-10) \cdot \frac{x^2+x-12}{x^2+11x+30}$

Solution: Flip the second term, factor, and cancel. The blue polynomial is a difference of cubes. Review the *Sum and Difference of Cubes* concept for how to factor this polynomial.

$$\begin{aligned}\frac{x^3-8}{x^2-6x+9} \div (x^2+3x-10) \cdot \frac{x^2+x-12}{x^2+11x+30} &= \frac{x^3-8}{x^2-6x+9} \cdot \frac{1}{x^2+3x-10} \cdot \frac{x^2+2x-15}{x^2+11x+30}\\ &= \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-3)}(x-3)} \cdot \frac{1}{\cancel{(x-2)}\cancel{(x+5)}} \cdot \frac{\cancel{(x+5)}\cancel{(x-3)}}{(x+5)(x+6)}\\ &= \frac{x^2+2x+4}{(x-3)(x+5)(x+6)}\end{aligned}$$

Guided Practice

Perform the indicated operations.

1. $\frac{a^5b^3c}{6a^2c^9} \div \frac{2a^7b^{11}}{24c^2}$
2. $\frac{x^2+12x-45}{x^2-5x+6} \div \frac{x^2+17x+30}{x^4-16}$
3. $(x^3+2x^2-9x-18) \div \frac{x^2+11x+24}{x^2-11x-24} \div \frac{x^2-6x-16}{x^2+5x-24}$

Answers

1. $\frac{a^5b^3c}{6a^2c^9} \div \frac{2a^7b^{11}}{24c^2} = \frac{a^5b^3c}{6a^2c^9} \cdot \frac{24c^2}{2a^7b^{11}} = \frac{24a^5b^3c^3}{12a^9b^{11}c^9} = \frac{2}{a^4b^8c^6}$

2.

$$\begin{aligned}\frac{x^2+12x-45}{x^2-5x+6} \div \frac{x^2+17x+30}{x^4-16} &= \frac{x^2+12x-45}{x^2-5x+6} \cdot \frac{x^4-16}{x^2+17x+30}\\ &= \frac{\cancel{(x+15)}\cancel{(x-3)}}{\cancel{(x-3)}\cancel{(x-2)}} \cdot \frac{(x^2+4)\cancel{(x-2)}\cancel{(x+2)}}{\cancel{(x+15)}\cancel{(x+2)}}\\ &= x^2+4\end{aligned}$$

3.

$$\begin{aligned}(x^3+2x^2-9x-18) \div \frac{x^2+11x+24}{x^2-11x+24} \div \frac{x^2-6x-16}{x^2+5x-24} &= \frac{x^3+2x^2-9x-18}{1} \cdot \frac{x^2-11x+24}{x^2+11x+24} \cdot \frac{x^2+5x-24}{x^2-6x-16}\\ &= \frac{(x-3)\cancel{(x+3)}\cancel{(x+2)}}{1} \cdot \frac{\cancel{(x-8)}(x-3)}{\cancel{(x+8)}\cancel{(x+3)}} \cdot \frac{\cancel{(x+8)}(x-3)}{\cancel{(x-8)}\cancel{(x+2)}}\\ &= (x-3)^2\end{aligned}$$

Problem Set

Divide the following expressions. Simplify your answer.

1. $\frac{6a^4b^3}{8a^3b^6} \div \frac{3a^5}{4a^3b^4}$
2. $\frac{12x^5y}{xy^4} \div \frac{18x^3y^6}{3x^2y^3}$
3. $\frac{16x^3y^9z^3}{15x^5y^2z} \div \frac{42xy^7z^2}{45x^2yz^5}$
4. $\frac{x^2+2x-3}{x^2-3x+2} \div \frac{x^2+3x}{4x-8}$
5. $\frac{x^2-2x-3}{x^2+6x+5} \div \frac{4x-12}{x^2+8x+15}$
6. $\frac{x^2+6x+2}{12-3x} \div \frac{6x^2-13x-5}{x^2-4x}$
7. $\frac{x^2-5x}{x^2+x-6} \div \frac{x^2-2x-15}{x^3+3x^2-4x-12}$
8. $\frac{3x^3-3x^2-6x}{2x^2+15x-8} \div \frac{6x^2+18x-60}{2x^2+9x-5}$
9. $\frac{x^3+27}{x^2+5x-14} \div \frac{x^2-x-12}{2x^2+2x-40} \div \frac{1}{x-2}$
10. $\frac{x^2+2x-15}{2x^3+7x^2-4x} \div (5x+3) \div \frac{21-10x+x^2}{5x^3+23x^2+12x}$