Chapter 2 Review

1. Write a summary of what you think are the important points of this chapter.

2. Joan, Henry, and Sam are heirs to an estate that includes a vacant lot, a boat, a computer, a stereo, and $11,000 in cash. Each heir submits bids as summarized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Joan</th>
<th>Henry</th>
<th>Sam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacant lot</td>
<td>$8,000</td>
<td>$7,500</td>
<td>$6,200</td>
</tr>
<tr>
<td>Boat</td>
<td>$6,500</td>
<td>$5,700</td>
<td>$6,700</td>
</tr>
<tr>
<td>Computer</td>
<td>$1,340</td>
<td>$1,500</td>
<td>$1,400</td>
</tr>
<tr>
<td>Stereo</td>
<td>$800</td>
<td>$1,100</td>
<td>$1,000</td>
</tr>
</tbody>
</table>

For each heir, find the fair share, the items received, the amount of cash, and the final settlement. Summarize your results in a matrix.

3. Anne, Beth, and Jay are heirs to an estate that includes a computer, a used car, and a stereo. Each heir submits bids for the items in the estate as summarized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Anne</th>
<th>Beth</th>
<th>Jay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer</td>
<td>$1,200</td>
<td>$900</td>
<td>$1,050</td>
</tr>
<tr>
<td>Car</td>
<td>$7,600</td>
<td>$7,400</td>
<td>$7,000</td>
</tr>
<tr>
<td>Stereo</td>
<td>$800</td>
<td>$600</td>
<td>$1,000</td>
</tr>
</tbody>
</table>

For each heir, find the fair share, the items received, the amount of cash, and the final settlement. Summarize your results in a matrix.
4. Lynn, Pauline, and Tim have just learned that they are the heirs to the estate of their recently deceased Uncle George. The only items of value in the estate are a rare guitar, a car, a kayak, and an expensive watch. Lynn, Pauline, and Tim submit bids as shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Lynn</th>
<th>Pauline</th>
<th>Tim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guitar</td>
<td>$1,500</td>
<td>$2,500</td>
<td>$2,200</td>
</tr>
<tr>
<td>Car</td>
<td>$6,000</td>
<td>$5,500</td>
<td>$5,500</td>
</tr>
<tr>
<td>Kayak</td>
<td>$700</td>
<td>$200</td>
<td>$600</td>
</tr>
<tr>
<td>Watch</td>
<td>$250</td>
<td>$400</td>
<td>$350</td>
</tr>
</tbody>
</table>

Lynn was always George’s favorite, and so his will states that Lynn should receive half of his estate and that Pauline and Tim should each receive a quarter. For each heir, find the fair share, the items received, the amount of cash, and the final settlement. Summarize your results in a matrix.

5. States A, B, and C have populations of 647, 247, and 106, respectively. There are 100 seats to apportion among them.

a. What is the ideal ratio?

b. Find the quota for each state.

c. Apportion the 100 seats among the three states by the Hamilton model.

d. What is the initial Jefferson apportionment?

e. Find the Jefferson adjusted ratio for each state.

f. Apportion the 100 seats by the Jefferson model.

g. What is the initial Webster apportionment?

h. Find the Webster adjusted ratio for each state.

i. Apportion the 100 seats by the Webster model.

j. What is the initial Hill apportionment?

k. Find the Hill adjusted ratio for each state.

l. Apportion the 100 seats by the Hill model.
Chapter 2 • Fair Division

m. Suppose the populations of the states change to 650, 255, and 105, respectively. Reapportion the 100 seats by the Hamilton model.

n. Explain why the results in part m constitute a paradox.

6. States A, B, C, and D have populations of 156, 1,310, 280, and 254, respectively. There are 20 seats to apportion among them. Two of the states have strong opinions about the model used, and the other two do not care. Determine which states care and explain why.

7. Discuss the theorem proved by Michel Balinski and H. Peyton Young. That is, what did they prove?

8. Arnold, Betty, and Charlie are dividing a cake in the following way.
   - Arnold divides the cake into what he considers six equal pieces.
   - The pieces are then chosen in this order: Betty, Charlie, Betty, Charlie, Arnold, Arnold.

   Who is guaranteed a fair share by his or her own assessment?

9. Four people have divided a cake into four pieces that each considers fair, and then a fifth person arrives. Describe a way to divide the four existing pieces so that each of the five people receives a fair share.

   In Exercises 10 and 11, collect and organize data into a table. Examine the data and conjecture a formula. Then use mathematical induction to prove that your formula is correct.

10. A basketball league schedules two games between each pair of teams. Find a formula for the number of games if the league has \( k \) teams.

11. In a set of concentric circles, a ring is any region that lies between any two of the circles. Find a formula for the number of rings in a set of \( k \) concentric circles.
12. On the basis of the enrollment in each of a high school’s courses, the administration must decide the number of sections that are offered. A number of factors affect the decision. For example, financial considerations require about 25 students in each section and a maximum of 300 sections for all courses. Develop a model the school might use to divide 300 sections fairly among all courses on the basis of the enrollment in those courses.

13. Discuss how fair division models you studied in this chapter might change to accommodate a situation in which the objects being divided are undesirable (i.e., the division of household chores among children).

**Bibliography**


