Chapter 3 Review

1. Write a summary of what you think are the important points of this chapter.

For Exercises 2 and 3, use the following matrices.

\[
A = \begin{bmatrix} 2 & 0 \\ 4 & 7 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 & 2 \\ 1 & 6 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & -2 \\ -1 & 0 \\ 3 & -4 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & -3 \\ 2 & -2 \\ 3 & -1 \end{bmatrix}
\]

2. a. How many elements does matrix \( B \) have?
   b. What is the value of \( C_{12} \)? Of \( C_{21} \)?

3. Find the value of each of the given expressions. If it is not possible, state the reason why.
   a. \( A + C \)
   b. \( C - B \)
   c. \((A + C) - D\)
   d. \(2A + D\)

4. Your math club is planning a Saturday practice session for an upcoming math contest. For lunch the students ordered 35 Mexican lunches, 6 bags of corn chips, 6 containers of salsa, and 12 six-packs of cold drinks.
   a. Write this information in a row matrix \( L \). Label your matrix.
   b. Interpret \( L_2 \) and \( L_4 \).
   c. Suppose that the club pays $4.50 per lunch, $1.97 per bag of corn chips, $2.10 for each container of salsa, and $2.89 for each six-pack of cold drinks. Use multiplication of a row and column matrix to find the total cost. Label your matrices.
5. A group of students is planning a retreat. They have contacted three lodges in the vicinity to inquire about rates. They found that Crystal Lodge charges $13.00 per person per day for lodging, $20.00 per day for food, and $5.00 per person for use of the recreational facilities. Springs Lodge charges $12.50 for lodging, $19.50 for meals, and $7.50 for use of the recreational facilities. Bear Lodge charges $20.00 per night for lodging, $18.00 a day for meals, and there is no extra charge for using the recreational facilities. Beaver Lodge charges a flat rate of $40.00 a day for lodging (meals included) and no additional fee for use of the recreational facilities.

a. Display this information in a matrix $C$. Label the rows and columns.

b. State the values of $C_{22}$ and $C_{43}$.

c. Interpret $C_{13}$ and $C_{31}$.

6. Mr. Jones has been shopping for a vacuum-powered cleaning system. He found one at Z-Mart and another model at Base Hardware. The Z-Mart system cost $39.50, disposal cartridges were 6 for $24.50, and storage cases were $8.50 each. At Base Hardware the system cost $49.90, cartridges were 6 for $29.95, and cases were $12.50 each.

a. Write and label a matrix showing the prices for the three items at the two stores.

b. Mr. Jones decided to wait and see if the prices for the systems would be reduced during the upcoming sales. When he went back during the sales, the Z-Mart prices were reduced by 10% and the Base Hardware prices were reduced by 20%. Construct a matrix showing the sale prices for each of the three items at the two stores.

c. Use matrix subtraction to compute how much Mr. Jones could save for each item at the two stores.

d. Suppose Mr. Jones is interested in purchasing the systems for himself and three of his friends. Use multiplication of a matrix by a scalar to find how much he would pay for each of the three items at the two stores at the sale prices.
7. The dimensions of matrices \( P, Q, R, \) and \( S \) are \( 3 \times 2, 3 \times 3, 4 \times 3, \) and \( 2 \times 3, \) respectively. If matrix multiplication is possible, find the dimensions of the following matrix products. If it is not possible, state why.
   a. \( QP \)
   b. \( RQ \)
   c. \( QS \)
   d. \( RPS \)

8. An artist creates plates and bowls from small pieces of colored woods. She currently has orders for five plates, three large bowls, and seven small bowls. Each plate requires 100 pieces of ebony, 800 pieces of walnut, 600 pieces of rosewood, and 400 pieces of maple. It takes 200 ebony pieces, 1,200 walnut pieces, 1,000 rosewood pieces, and 800 pieces of maple to make a large bowl. A small bowl takes 50 pieces of ebony, 500 walnut pieces, 450 rosewood pieces, and 400 pieces of maple.
   a. Write a row matrix showing the current orders for this artist’s work.
   b. Construct a matrix showing the number of pieces of wood used in an individual plate or bowl.
   c. Use matrix multiplication to compute the number of pieces of each type of wood the artist will need for the plates and bowls that are on order.
   d. Suppose it takes the artist 3 weeks to fashion a plate, 4 weeks to make a large bowl, and 2 weeks to complete a small bowl. Use matrix multiplication to show how long it will take the artist to fill all the orders for plates and bowls.

9. Tonya has money invested in three sports complexes in Smith City. Her return (annual) from a $50,000 investment in a tennis club is 8.2%. She receives 6.5% from a $100,000 investment in a golf club and 7.5% on a $75,000 investment in a soccer club. Use matrix multiplication to find Tonya’s income from her investments for one year. Label your matrices.

10. Three music classes at Central High are selling candy as a fundraiser. The number of each kind of candy sold by each of the three classes is shown in the following table.
### Chapter 3 • Review

#### 11. Use the following matrices to find the value of each of the given expressions. If the expression is not defined, give the reason.

- **A** = \[
\begin{bmatrix}
2 & 0 \\
4 & 7 \\
-1 & 3
\end{bmatrix}
\]
- **B** = \[
\begin{bmatrix}
4 & -1 & 2 \\
1 & 6 & -3
\end{bmatrix}
\]
- **C** = \[
\begin{bmatrix}
1 & 2 \\
-1 & 3
\end{bmatrix}
\]
- **D** = \[
\begin{bmatrix}
2 & 1 & 0
\end{bmatrix}
\]
- **E** = \[
\begin{bmatrix}
4 & 3
\end{bmatrix}
\]

- a. **AB**
- b. **BA**
- c. **CA**
- d. **DA + E**

#### 12. Write the transpose \((A^T)\) of matrix **A**, where

\[
A = \begin{bmatrix}
4 & 2 & 6 \\
5 & 1 & 3
\end{bmatrix}
\]

#### 13. Let matrix

\[
M = \begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix}
\]

- a. Calculate **M**\(^2\), **M**\(^3\), and **M**\(^4\).
- b. Predict the components of **M**\(^5\) and check your prediction.
- c. Generalize to **M**\(^n\), where **n** is a natural number.
- d. Prove your conjecture in part c using mathematical induction.
- e. Repeat parts a, b, c, and d for the matrix

\[
M = \begin{bmatrix}
1 & 0 \\
2 & 3
\end{bmatrix}
\]
14. Complete the following statement: If a square matrix \( A \) has an inverse \( A^{-1} \), then the product \( AA^{-1} \) = the ______ matrix \( I \), where \( I \) is a ________.

15. Which of the following matrices are inverses of each other? Explain your answers.
   a. \[
   \begin{bmatrix}
   -1 & 3 \\
   2 & -5
   \end{bmatrix}
   \text{ and } \begin{bmatrix}
   5 & 3 \\
   2 & 1
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   1 & 0 \\
   0 & 1
   \end{bmatrix}
   \text{ and } \begin{bmatrix}
   1 & 0 \\
   0 & 1
   \end{bmatrix}
   \]
   c. \[
   \begin{bmatrix}
   2 & 1 & 0 \\
   3 & 2 & 1
   \end{bmatrix}
   \text{ and } \begin{bmatrix}
   1 & -1 \\
   -1 & 2 \\
   -1 & 2
   \end{bmatrix}
   \]

16. The students at Central High are planning to hire a band for the prom. Their choices are bands \( A \), \( B \), and \( C \). They survey the Sophomore, Junior, and Senior classes and find the following percentages of students (regardless of sex) prefer the bands,

\[
\begin{align*}
\text{10th} & \quad \text{11th} & \quad \text{12th} \\
A & \quad 20\% & \quad 35\% & \quad 40\% \\
B & \quad 30\% & \quad 30\% & \quad 25\% \\
C & \quad 50\% & \quad 35\% & \quad 35\%
\end{align*}
\]

The student population by class and sex is:

\[
\begin{array}{ccc}
\text{Male} & \text{Female} \\
\text{10th} & 235 & 225 \\
\text{11th} & 205 & 215 \\
\text{12th} & 175 & 190 \\
\end{array}
\]

Use matrix multiplication to find:

a. The number of males and females who prefer each band.

b. The total number of students who prefer each band.
17. The characteristics of the female population of a herd of small mammals are shown in the following table.

<table>
<thead>
<tr>
<th>Age Groups (months)</th>
<th>0–4</th>
<th>4–8</th>
<th>8–12</th>
<th>12–16</th>
<th>16–20</th>
<th>20–24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Birth Rate</td>
<td>0</td>
<td>0.5</td>
<td>1.1</td>
<td>0.9</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>Survival Rate</td>
<td>0.6</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
<td>0.6</td>
<td>0</td>
</tr>
</tbody>
</table>

Suppose the initial female population for the herd is given by

\[ P_0 = [22 \ 22 \ 18 \ 20 \ 7 \ 2]. \]

a. What is the expected lifespan of this mammal?
b. Construct the Leslie matrix for this population.
c. Determine the long-term growth rate for the herd.
d. Suppose this mammal starts dying off from overcrowding when the total female population for the herd reaches 520. How long will it take for this to happen?

**Bibliography**


